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SPATIALLY AUTOCORRELATED ERRORS IN ORIGIN-DESTINATION MODELS: A NEW SPECIFICATION APPLIED TO AGGREGATE MODE CHOICE

by

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RESUME

Afin de prendre en compte l'autocorrélation spatiale dans les résidus d'une régression linéaire visant à expliquer des données de flux, on postule une formulation autorégressive spatiale d'ordre 1 (SAR(1)). Selon cette formulation, le terme d'erreur d'une observation se décompose en une somme pondérée des autres erreurs du modèle et un terme purement aléatoire. Les pondérations nouvelles utilisées sont des formes fonctionnelles paramétriques qui mesurent la proximité entre les origines et les destinations des flux. Ces paramètres sont estimés conjointement avec ceux de la régression par la méthode du maximum de vraisemblance. La procédure est appliquée à un modèle de parts bimodal logit agrégé pour expliquer des déplacements pour motif travail à l'heure de pointe du matin à Winnipeg au Canada.

Mots clés: moindres carrés généralisés, autocorrélation spatiale, flux origine-destination, partage modal, logit, déplacements à motif travail, Winnipeg.

ABSTRACT

In this study, we use a first order spatial autoregressive (SAR(1)) formulation to model the correlation among the errors of a linear demand equation that explains origin-destination (O-D) flows. The SAR(1) process splits the error term for each observation into a weighted sum of all the other errors and a purely random noise. The weights are new parametric functional forms defined to measure the proximity between origins and destinations of flows. The parameters of these weights, along with the other parameters of the model, are estimated by the method of Maximum Likelihood. We apply the technique to an aggregate binary Logit share model that explains peak a.m. trips to work in Winnipeg, Canada.

Keywords: Generalized Least Squares, Spatial Autocorrelation, Origin-Destination Flows, Logit, Mode Choice, Work Trips, Winnipeg.

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1 INTRODUCTION : THE EXPLANATION OF O-D FLOWS

To explain passengers' interzonal (O-D) travel flows, transportation researchers often use a linear demand formulation. This can be written as:

$$y_{r,s}^{m} = \sum_{k=1}^{K} x_{r,s}^{k} \beta_k + \varepsilon_{r,s}^{m}, \quad r = 1,2,\ldots,R \; , \; s = 1,2,\ldots,S \; , \; m = 1,2,\ldots,M \quad , \tag{1}$$

where the dependent variable denotes the travel demand y from zone r to zone s by mode m. One of the earliest examples of this type of modeling in urban context is the model by Domencich, Kraft and Valette (1968). A more recent example may be found in Algers (1984).

The explanatory variables usually considered include: i) level of service (LOS) variables specific to mode m, for instance travel cost, on-vehicle travel time, out-of-vehicle travel time; ii) LOS variables of competing modes; iii) variables capturing generation and attraction characteristics of the zones at the origin and at the destination of the trips. Examples of these last variables are : parking cost at the destination, average income or average number of cars per household at the origin, etc....

Often, because of omission of variables or because of lack of data, the model cannot capture all relevant factors related to the geographic structure. For instance, a shopping trip to one area may be explained by the absence of other shops with comparable characteristics in neighboring areas. Similarly, if two *contiguous* destinations have equivalent shopping facilities (shopping malls, for instance), these destinations may be close substitutes for the shoppers of the

neighboring zones. The fact of omitting these factors affects the residual errors and may induce spatial autocorrelation. In such cases, estimating the model by the Ordinary Least Squares (OLS) method is inefficient.

To model the structural relationships between residual errors, notably in linear travel flow demand models, we suggest below a spatial autoregressive of order 1 (SAR(1)) framework. Some general properties of SAR(1) error formulations are reviewed in the next section. We then apply the proposed methodology to an aggregate mode share demand model currently in use in Winnipeg. Our principal goal in this paper is to show how a model that seems to perform well can still be improved by taking into account the spatially autocorrelated structure of the errors. The estimation procedure is described in the Appendix.

2 AN AGGREGATE LINEAR O-D MODEL WITH SAR(1) ERRORS

In the SAR(1) framework, the error term ε_i is expressed as a weighted sum of the other errors of the regression plus a purely random element. To simplify the notation, the mode superscript has been dropped and the subscripts r,s have been changed to $i,(i = 1,..,N)$, where N is the number of observations in the sample. Assuming a normal distribution for the errors, the origin-destination trip model with SAR(1) errors can be written as :

$$y_i = \sum_{k=1}^{K} x_i^k \beta_k + \varepsilon_i \qquad i = 1,\ldots,N \tag{2}$$

$$\varepsilon_i = \rho_1 \sum_{j=1}^{N} w_{ij}\varepsilon_j + u_i, \text{ where } u_i \text{ is i.i.d. } N(0,\sigma^2) \quad \forall i, \text{ and } w_{ii} = 0. \tag{3}$$

In matrix notation, Equations (2) and (3) can be written as:

$$y = X\beta + \varepsilon$$
$$\varepsilon = \rho_1 W\varepsilon + u, \quad u \sim N(0,\sigma^2 I_N). \tag{4}$$

u is a $(N \times 1)$ vector of i.i.d. normally distributed random variables u_i with mean 0 and variance σ^2. X is a $(N \times K)$ fixed matrix of observations and β is a $(K \times 1)$ vector of unknown coefficients. ε_i represents the number of unexplained trips. W denotes the $(N \times N)$ spatial lag operator matrix with elements w_{ij}, which are estimable functional forms specified by the modeler to describe the spatial pattern among the errors. Using a general notation, the function w_{ij} can be written as:

$$w_{ij} = g(V_{ij}^1, V_{ij}^2, \ldots, V_{ij}^L; \theta_1, \theta_2, \ldots, \theta_H) \text{ with } w_{ii} = 0 \quad \forall i, \tag{5}$$

where there are H parameters θ_h, $(h = 1,2,\ldots,H)$ and L variables V_{ij}^l, $(l = 1,2,\ldots,L)$. The numbers of parameters and variables are assumed to differ in order to allow for non-linear specifications.

Ideally, one could have specified an unconstrained dynamic model in which each explanatory variable of the model would have its proper spatial structure. This would allow, for example, when considering a given flow $r \to s$, to include cross price and service effects from other O-D pairs in addition to the own effects associated with r and s. However, in a multivariate formulation, such an approach would be much too complicated to implement. Imposing spatial correlation among the residual errors can be seen as a more tractable approximation to this more complete specification. Furthermore, the correlations among errors may also reflect the specifics of the particular zoning scheme used in models that are based on zonal aggregates assigned to centroids.

In the present study, the function w_{ij} must be defined in such a way that it can measure the *proximity* between two unexplained *flows* ε_i and ε_j. To obtain a simple function with this property, in travel flow formulations, we use a mixture of distances between pairs of origins and pairs of destinations of trips. Details will be given in Section 2.1.

The specification of the w_{ij}'s in standard spatial models where each observation of the dependent variable is not a flow between zones, as in this paper, but pertains only to a single zone n , ($n \in \{1,2,...,N\}$) is a rather straightforward exercise. Spatial studies that use a SAR(1) error process often define the contiguity structure through a Boolean contiguity matrix. In that case, $w_{nm} = 1$ if n and m are contiguous and $w_{nm} = 0$ otherwise. See, for instance, Cliff and Ord (1981), Blommestein (1983) or Bivand (1984). Alternatively, one can also define a contiguity matrix with elements $w_{nm} = (d_{nm})^{-\alpha}$ with $\alpha > 0$ where d_{nm} is the distance between n and m. For a given value of α, the greater the distance between two areas, the less are the corresponding errors related.

The parameter ρ_1 is a spatial autocorrelation coefficient which measures the degree of linear dependency between the ε_i's and their corresponding spatial neighbors. When w_{ij} is constrained to be non-negative, ρ_1 depicts the sign of the correlation as well. The interpretation and the sign of ρ_1 depends on the definition of proximity embodied in the matrix W. For the specifications of W discussed in the previous paragraph, a $\rho_1 > 0$ implies that errors of the same sign are geographically grouped together. Alternatively, if we had specified $w_{ij} = (P_n P_m)^{\alpha}$, $\alpha > 0$, where P_n and P_m are the populations in area n and m respectively, we would find the strongest correlations among the errors of areas containing the largest populations. Other interpretations of ρ_1 in travel demand formulations will be given in the next section.

Brandsma and Ketellaper (1979) suggested an alternative methodology for modeling travel flows with spatially autocorrelated errors. Their model, which is specifically designed to explain *inter*-urban flows can easily be adapted to model urban *inter*-zonal flows. After some simple transformations, their model can be expressed as a linear model with SAR(2) errors :

$$\varepsilon_i = \rho_1 \sum_{j=1}^{N} w_{1,ij}\varepsilon_j + \rho_2 \sum_{j=1}^{N} w_{2,ij}\varepsilon_j + u_i, \quad u_i \sim N(0,\sigma^2) \quad \forall i,$$

where $w_{1,ij}$ and $w_{2,ij}$ define two different Boolean operators capturing destination and origin effects, respectively. Extensions of this model to allow for estimable $w_{1,ij}$'s and $w_{2,ij}$'s as well as a discussion on the applicability of this approach to inter-zonal flow models can be found in Chapter 8 and Chapter 9 of Bolduc (1985).

The alternative approach that we propose below turns out to be much easier to handle and interpret. As we will see, we can generate fairly complex patterns of correlation with very simple w_{ij} functions.

Another interesting approach for dealing with correlated errors would consist in modeling the elements of the errors' correlation matrix directly. Early basic modeling efforts in this direction can be found in Fisher (1971). Attempts with general functional forms in the context of a zone specific model were also made by Bolduc (1985) where, using the same notation as in the zone specific examples previously discussed, one finds for instance:

$$r_{nm} = \frac{1 - \exp^{-\phi(z_{nm}'\alpha)}}{1 + \exp^{-\phi(z_{nm}'\alpha)}},$$

where $r_{\cdot\cdot}$ is the correlation coefficient between $\varepsilon_{\cdot}$ and $\varepsilon_{\cdot\cdot}$, $\phi(\cdot)$ is a generalized function to be defined by the modeler, $z_{\cdot\cdot}$ is a vector of explanatory variables and α is a vector of parameters. Although interesting results were obtained for very simple inverse functions of distance, this very appealing methodology does not, contrary to the SAR(1) approach, guarantee the positive definiteness of the fitted correlation matrix.

2.1 A Functional Form for the Weights

A simple parametric form that measures proximity between unexplained flows from r to s and from l to t is:

$$w_{ij} = (d_{r,l} + d_{s,t})^{-\theta_1} + (d_{r,t} + d_{l,s})^{-\theta_2}, \quad \text{with} \quad w_{ii} = 0 \quad \forall i. \tag{6}$$

This function[1] takes into consideration direct and cross distance effects between two origins and two destinations.

For a given pair of observations, say $r \to s$ and $l \to t$, the first of the two components in equation (6), the direct effect, refers to the distances between the two origins of the trips r and l and the two destinations s and t. For a given value of $\theta_1 > 0$, small distances between the origins and/or the destinations induce a large weight w_{ij}. This first term is used to account for a mixture of positive and negative relationships between individual flows originating and terminating in specified areas. The expectations about the sign dominating the relationship (i.e. the sign of ρ_1) depend on the trip purpose under consideration. Take for example shopping trips. For a given

1 One could also conceive of a multiplicative form.

trip origin r, the presence of similar shops in two neighboring areas at the destination induces, because of competition, a negative relationship between the flows, and thus most likely between the unexplained flows heading to s and t.

In work trips models, however, the major relationships between residual flows are expected to be positive at both ends of the trips. Let us start with the effects at the destination. Similar industries or firms are very often geographically grouped in the Central Business District (CBD) or in the same industrial areas. Unexplained work trips to neighboring areas belonging to the CBD, for instance, are expected to be of the same nature. In a like manner, unexplained trips for work purposes from a given zone to contiguous residential areas with similar characteristics are likely to fall in the same categories. This suggests a positive relationship between residual flows at the destination. It is hard to believe that the negative relationship caused, for example, by the employment competition between firms at the destination would be a factor more important that the one just described. Consider now the possible relationships at the origin. Since one of the main determinants of residential location is proximity to work, unexplained home-based trips to work at the same destination originating from contiguous residential areas are expected to have a certain degree of commonality. Here again, the average correlation between the errors is expected to be positive.

The second term in the RHS of equation (6) represents the cross distance effect between the origins and the destinations. Our formulation implies that for a given $\theta_2 > 0$, everything else being equal, the higher those distances are, the smaller is the weight w_{ij}.

Before we apply the SAR(1) error formulation to origin-destination type data, we summarize below general results concerning SAR(1) error processes. Details can be found in Bolduc (1987).

2.2 Summary of the SAR(1) Formulation

- Stable SAR(1) Error Models:

A SAR(1) model is *stable* if the value of ρ_1 is such that the infinite sum:

$$\varepsilon = (I - \rho_1 W)^{-1} u = \sum_{i=0}^{\infty} \rho_1^i W^i u \tag{7}$$

converges. From stability of Markov Processes theory, the values of ρ_1 for which this condition holds is known to satisfy:

$$| \rho_1 | < 1 / | \lambda_{MAX}(W) | ,$$

where $| \lambda_{MAX}(W) |$ denotes the largest eigenvalue λ of matrix W, in absolute value.

We list two normalizations on W (a matrix which may or may not be symmetric) such that the corresponding SAR(1) error model is *stable* when $\rho_1 \in (-1,1)$. These versions of the SAR(1) error model are called here *normalized SAR(1) error* models. The elements of the first normalized matrix are defined by:

$$w_{ij} = \frac{w_{ij}^*}{\sum_{j=1}^{N} w_{ij}^*}, \quad \text{with} \quad w_{ii} = 0 \quad \forall i \quad \text{and} \quad \sum_{i=1}^{N} w_{ij} = 1 \quad \forall i, \tag{8}$$

where W^* denotes the original weighting matrix on which the normalization is applied. The normalised W matrix defined by (8) can never be symmetric. By contrast, the matrix W obtained from a symmetric W^* matrix retains the property of symmetry if the normalization rule is:

$$w_{ij} = \frac{w_{ij}^{*}}{\sqrt{\left(\sum_{i}^{N} w_{ij}^{*}\right)}\sqrt{\left(\sum_{j}^{N} w_{ij}^{*}\right)}}, \quad \text{with} \quad w_{ii} = 0 \quad \forall i. \tag{9}$$

To show the stability over $\rho_1 \in (-1,1)$ of the SAR(1) representations defined by (8) or (9), first note that the W matrices defined by (8) and (9) have the same eigenvalues, and then use the following well-known relation :

$$|\lambda| \leq \max_{i} \sum_{j} |w_{ij}| .$$

In our empirical application, we will use the symmetric normalized form (9) because our original contiguity matrix W is symmetric and because we found a symmetric contiguity matrix to be more convenient in term of computer memory space.

The use of normalizations (8) or (9) is recommended because: i) they produce a globally concave likelihood function over the open unit interval $(-1,1)$ of ρ_1; ii) in the case of a well identified continuous parametric structure in the w_{ij}'s, when ρ_{1_o} [the true value of ρ_1] belongs to a compact subspace of $|\rho_1| < 1$, the Maximum Likelihood procedure gives consistent and asymptotically normal estimators. Properties of consistency and of asymptotic normality of the ML estimates of SAR(1) error models with general functional forms are considered in Bolduc (1987). Since the functional form proposed in (6) is a special case of the forms considered there, these two properties are inferred to hold in the present model.

- OLS Estimation of ρ_1

Whittle (1954) and Ord (1975) proved that the OLS estimator of ρ_1 is inconsistent when the structure of dependency is multidirectional. This problem is identical to the problem of OLS

estimation in linear simultaneous equations models. In recursive systems (where W is an upper or a lower triangular matrix), however, the estimator is consistent. Time Series models constitute examples of recursive systems. The typical example of a recursive model in spatial processes is the wave model described in Haining (1978). To produce a consistent estimation of ρ_1 in SAR(1) models with multidirectional dependencies, Ord (1975) proposed an iterative Cochrane - Orcutt (1949) procedure where ρ_1 is estimated by the method of Maximum Likelihood (ML) (for an equivalent full ML procedure see: Hepple, 1976). Except for Bodson and Peeters (1975) who calibrated the weights in a step prior to the estimation, the functions w_{ij} were always fixed before estimation until Bolduc (1985) developed a general procedure that jointly estimates by the method of maximum likelihood, the parameters contained in the w_{ij}'s along with the other parameters of the model. This same approach is used here to model a linear travel flow demand with autocorrelated errors. In order to generalize the model and to relax some of the constraints, we will also estimate a SAR(2) formulation.

3 EMPIRICAL APPLICATION

3.1 An Urban Mode Choice Model

- Aggregate Binary Logit Share Specification

Laferrière (1981), using a slightly modified version of a model originally specified by Florian (1979), considered a two-mode share model formulation for automobile and bus where the shares, denoted respectively by p_A and p_B, are expressed in a Logit form :

$$p_A = \frac{e^{V_A}}{e^{V_A} + e^{V_B}}$$

$$p_B = \frac{e^{V_B}}{e^{V_A} + e^{V_B}} \qquad (10)$$

where V_A and V_B are mode specific linear in parameters functions containing characteristics of the modes and socioeconomic variables describing the travelers associated with the origin and the destination. This is our application model.

For simplicity, the r,s subscripts have been ignored. The transformation $y = \ln(p_B/p_A)$ and the addition of a residual error term lead to the following linear model:

$$y = X\beta + u \qquad (11)$$

where $X\beta = V_B - V_A$. Since the previous transformation is such that $y \in (-\infty,\infty)$, if specification (11) is true, the OLS estimation method has some optimal properties when u is a vector of i.i.d. components[2].

- Data Used

The data used in the testing of this model were collected in the city of Winnipeg Canada during the summer 1976. The sample comprises twenty percent of the total population. Only observations pertaining to peak a.m. home origin based trips to work were retained in the present study. Sample means of observed automobile and bus shares are .635 and .365, respectively.

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2 This derivation can also be made using a disaggregate behavioral approach. For details see Chapter 4 of Ben-Akiva and Lerman (1985), pp. 95-97.

All the information was aggregated at the level of 147 zones. Most of the level of service variables were calculated using different procedures of assignment that aim to minimize the travel time. For estimation, we kept only the observations for which the total of the flows by car and bus is larger than 60. The final estimation sample has 211 observations. The results presented in the following sections were estimated on a CDC 875 mainframe computer at the Université de Montréal.

- *Model Specification*

The estimated model is written:[3]

$$\ln(p_B/p_A) = \beta_0 + \beta_1 \text{TZIMP1} + \beta_2 \text{AUTIM1} + \beta_3 \text{TAUT} \\ \beta_4 \text{OINC} + \beta_5 \text{OCAR} + \beta_6 \text{MEN1} + u. \qquad (12)$$

This model specification uses a combination of LOS variables (TZIMP1, AUTIM1 and TAUT) and socioeconomic variables (OINC, OCAR and MEN1). Their definitions, as well as their respective sample means, follow:

- TZIMP1 : Travel time (in minutes) by bus, which is the sum of the on-vehicle travel time, the waiting, the access and the egress time. (Mean: 76.91 minutes.)
- AUTIM1 : Travel time by car, expressed in minutes. (Mean: 20.90 minutes.)

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3 V_B and V_A are postulated to be as follows:

$$V_B = (\beta_{00} + \beta_3 \text{TFBUS}) + \beta_1 \text{TZIMP1} + \beta_4 \text{OINC} + \beta_5 \text{OCAR} + \beta_6 \text{MEN1}$$

$$V_A = \beta_2^* \text{AUTIM1} + \beta_3^* \text{TAUT}$$

with $\beta_2 = -\beta_2^*$ and $\beta_3 = -\beta_3^*$. Note also that $\beta_0 = \beta_{00} + \beta_3 \text{TFBUS}$. The time variables are assumed to be mode-specific.

- TAUT : Travel cost by car defined as:(.058 × distance) + (parking cost at the destination). (Mean: 0.53 dollars.)
- OINC : Average income of households at the zone of origin of the trip. (Mean: 16355 dollars.)
- OCAR : Average number of cars per household at the origin. (Mean: 0.96 cars.)
- MEN1 : Proportion of males traveling on the O-D segment. (Mean: 0.60.)

The travel cost by bus, which is fixed at 25 cents for everyone, is implicitly included in the constant term with a coefficient assumed equal to β_3. Since (12) is defined as the ratio of bus to auto shares, the following relationships are anticipated. The effect of the auto (bus) LOS variables is anticipated to be positive (negative) respectively. The bus mode share is expected to be inversely related to the number of cars and the level of income at the origin. The variable MEN1 represents the inclination of males to use their car for trips to work.

3.2 Ordinary Least Squares Results

OLS estimates are presented in Column 1 of Table 1. All the coefficient estimates have the expected sign. AUTIM1 and OINC are, however, not significantly different from zero at a reasonable level of significance. With predicted shares of .673 and .327 compared with the observed shares of .635 and .365 for auto and bus, respectively, the OLS estimates performed quite well in this respect. Details on the computations of the predicted shares found in Table 2 are given in the next paragraph.

Mode share elasticities and percentage points corresponding to the OLS estimates are presented in Column 1 of Table 2. The computed elasticities all have the anticipated signs and values. The auto share elasticities are small and uniformly about half the bus share elasticities. Percentage points are defined as elasticities multiplied by the average mode share. For example, the percentage point value of .288 for travel time by bus is computed as: .427 x .673 or .880 x

.327 where .673 and .327 are the average of the fitted mode shares for auto and bus respectively. Percentage points therefore express the number of market share points gained (or lost) due to a one percent change in the explanatory variable considered[4]. The formulas are:

$$\text{Elasticity :} \quad \hat{\eta}_{mk} = \hat{\beta}_k \bar{X}_k (1 - \hat{p}_m) , \quad m = \text{auto, bus} , \; k = 1,\ldots,K$$

$$\text{Percentage points :} \quad \hat{v}_{mk} = \hat{\eta}_{mk} \hat{p}_m$$

where :

$$\hat{p}_m = \frac{1}{N} \sum_{i=1}^{N} \hat{p}_{im} \quad \text{and} \quad \hat{p}_{im} = \frac{e^{\hat{V}_{im}}}{e^{\hat{V}_{im'}} + e^{\hat{V}_{im}}} , \; m' \neq m.$$

4 Alternatively, one could say, referring to the example above, that when travel time by bus doubles, the average probability of using the bus decreases by .288, or the market share decreases by 28.8 percentage points.

Table 1

Logit model of the ratio of bus trips to automobile trips by origin-destination pair in Winnipeg, 1976:

Coefficients and t-statistics.

VARIABLES		OLS	GLS SAR(1)	GLS SAR(2)
Travel time by bus	TZIMP1	-.017 (-4.863)	-.009 (-2.047)	-.012 (-2.885)
Travel time by car	AUTIM1	.003 (.255)	.009 (.827)	.013 (1.242)
Travel cost by auto	TAUT	1.416 (4.941)	.760 (2.504)	.792 (2.592)
Income at origin	OINC	.000005 (.255)	.000007 (.355)	.00001 (.578)
Cars per household at origin	OCAR	-1.903 (-4.080)	-1.453 (-2.933)	-1.672 (-3.693)
Proportion of men on O-D pair	MEN1	-1.769 (-4.994)	-1.860 (-5.430)	-1.948 (-5.689)
	CONSTANT	2.592 (9.218)	-.586 (-.710)	1.513 (3.575)
Other parameters				
	$\hat{\rho}_1$		.979 (67.410)	.499 (1.955)
	$\hat{\rho}_2$			.703 (2.249)
	$\hat{\theta}_1$		1.142 (8.653)	
	$\hat{\theta}_2$		2.278 (1.525)	
	$\hat{\lambda}_1$			1.812 (1.771)
	$\hat{\lambda}_2$			2.212 (1.522)
Log likelihood		60.740	69.347	67.482
Number of observations		211	211	211

Table 2

Logit model of the ratio of bus trips to automobile trips by origin-destination pair in Winnipeg, 1976:

A: Elasticities and percentage points.*

VARIABLES		OLS	GLS SAR(1)	GLS SAR(2)
Travel time by bus	TZIMP1	-0.880 0.428 *0.288*	-0.457 0.235 *0.155*	-0.576 0.324 *0.207*
Travel time by car	AUTIM1	0.042 -0.021 *0.014*	0.124 -0.064 *0.042*	0.179 -0.101 *0.065*
Travel cost by bus	TFBUS	-0.238 0.116 *0.078*	-0.125 0.065 *0.043*	-0.126 0.071 *0.046*
Travel cost by auto	TAUT	0.508 -0.247 *0.166*	0.267 -0.138 *0.091*	0.269 -0.152 *0.097*
Income at origin	OINC	0.055 -0.027 *0.018*	0.076 -0.039 *0.026*	0.126 -0.071 *0.045*
Cars per household at origin	OCAR	-1.231 0.598 *0.403*	-0.922 0.475 *0.313*	-1.029 0.579 *0.370*
Proportion of men on O-D pairs	MEN1	-0.725 0.352 *0.237*	-0.747 0.385 *0.254*	-0.759 0.427 *0.273*

B: Mean shares.

mean share :	bus: .365 auto: .635	.327 .673	.340 .660	.360 .640

* The three numbers given are, successively, the bus share elasticity, the auto share elasticity and, in italics, the absolute value of percentage points. The percentage points, with respect to a mode, have the same sign as the corresponding elasticities.

3.3 SAR(1) Estimates

In this section, we present the solution of the Mode Choice Model with errors assumed to be generated by the SAR(1) process described in Section 2.1. The spatial process is postulated to be stable through a symmetric normalization (9) of the following weights[5,6]:

$$w_{ij}^{*} = (d_{r,j} + d_{s,j})^{-\theta_1} + (d_{r,j} + d_{l,s})^{-\theta_2} \quad \text{with} \quad w_{ii}^{*} = 0 \quad \forall i$$

Parameter estimates are presented in Column 2 of Table 1. In addition to the fact that the constant term is now unsignificant, the major changes compared to the OLS estimates are for variables AUTIM1 and TAUT. The estimated values for AUTIM1 and TZIMP1 seem to indicate that the travel time variable should be postulated to be generic. This would mean that 1 minute of travel time by car or by bus has the same value to the travelers, a constraint often imposed by transportation demand modelers. The auto fare effect TAUT decreased significantly to half of its OLS value.

The spatial correlation coefficient $\hat{\rho}_1$ is significantly different from zero and positive. Since we are modeling trips to work, this result is expected. Using the standard error as a guide, the true value of θ_2 cannot be ascertained with precision. However, judging from the point

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5 In the case of Winnipeg, the distances used are shortest distances between centroids. The original matrix except for one-way street effects is symmetric. The matrix actually used was made completely symmetric by taking averages of shortest distances in both directions. As pointed out above, the properties of the maximum likelihood estimates do not however depend on the symmetry of W.

6 Note that this weighting scheme has shortcomings. It is not invariant with respect to changes in units of measurements. Furthermore, it would be inappropriate for samples containing flows between modes going in opposite directions since, for these observations, we would have :

$(d_{rr} + d_{ls})^{-\theta_2} \to \infty$ for $\theta_2 > 0$.

Although this weighting scheme may be a satisfactory approximation for the present study, more sophisticated schemes that avoid these shortcomings could clearly have been conceived.

estimates, since distances are all greater than one, the estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ suggest that the direct contiguity effect at the origin and at the destination is more important than the cross distance effect since a larger value of θ_2 implies a smaller impact of the cross distance term. This is not surprising, because travel time and cost variables already account for distance between the origin and the destination, but do not account for competition between close origins and destinations[7]. Comparable GLS results were obtained with the alternative form:

$$w_{ij}^{*} = 1/\left[(d_{r,j}+d_{s,s})^{\theta_1}+(d_{r,j}+d_{l,s})^{\theta_2}\right] \quad \text{with} \quad w_{ii}^{*}=0 \quad \forall i.$$

The estimates for the θ_i's in this case are : $\hat{\theta}_1 = 1.538$ and $\hat{\theta}_2 = -1.373$. Under this specification of the w_{ij}'s, a negative value for θ_2 implies that the cross distance effect is not important in explaining the contiguity structure.

To predict mode shares of a given O-D pair, given the shares associated with other observations in the sample, we use :

$$\hat{p}_{Bi} = \frac{e^{\hat{y}_i}}{(1+e^{\hat{y}_i})} \text{ and } \hat{p}_{Ai} = 1-\hat{p}_{Bi}$$

where $\hat{y}_i$ is defined as follows:

$$\hat{y}_i = \sum_{k=1}^{K} x_i^k \hat{\beta}_k + \hat{\rho}_1 \sum_{j=1}^{N} w_{ij}(\hat{\theta})\left(y_i - \sum_{k=1}^{K} x_i^k \hat{\beta}_k\right) \tag{13}$$

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7 By structure, demand or mode choice models of the type used here make the flow between r and s depend on prices and service levels between r and s only and do not contain similar terms for r and t.

with $\hat{\beta}$, $\hat{\theta}$, and $\hat{\rho}_1$ denoting ML estimates. Under the hypothesis that ρ_1 and θ are known and conditional on the vector $(y_1,\ldots,y_{i-1},y_{i+1},\ldots y_N)$, this is the minimum mean squared error linear unbiased predictor for $y_i = \ln(p_{Bi}/p_{Ai})$ that would correspond to Equation (4) which can also be written as :

$$y = X\beta + \rho_1 W(\theta)(y - X\beta) + u.$$

With predicted shares of .66 and .34 for auto and bus respectively, the SAR(1) approach leads to slightly better predictions than in the OLS case. The elasticities, as well as the percentage points, with fitted shares computed following Equation (13), are presented in Table 2.

3.4 SAR(2) Estimates

In the previous section, we assumed that the direct contiguity effect at the origin was the same as that at the destination of the trips. We now relax this constraint and postulate a SAR(2) formulation[8], which can be written as :

$$\begin{aligned} y &= X\beta + \varepsilon \\ \varepsilon &= \rho_1 W_1 \varepsilon + \rho_2 W_2 \varepsilon + u, \quad u \sim N(0,\sigma^2 I_N), \end{aligned} \tag{14}$$

where W_1 and W_2 are normalized according to (9). The following specification allows the factors at the origin and at the destination of the trips to have different direct effects on the correlation pattern among the errors. For a given observation i, it is expressed as:

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8 Theoretical properties of the SAR(2) model were not derived, but grid searches of a version with Boolean contiguity matrices showed a well behaved likelihood function over the domain defined by $\rho_1 \in (-1,1)$ and $\rho_2 \in (-1,1)$.

$$\varepsilon_i = \rho_1 \sum_{j=1}^{N} w_{1,ij}\varepsilon_j + \rho_2 \sum_{j=1}^{N} w_{2,ij}\varepsilon_j + u_i, \quad u_i \sim N(0,\sigma^2) \quad \forall i, \tag{15}$$

where

$$w_{1,ij} = d_{r,s}^{-\lambda_1} \quad \text{with} \quad w_{1,ii} = 0 \quad \forall i \text{ and } w_{1,ij} = 0 \text{ if } r = l$$

and

$$w_{2,ij} = d_{s,t}^{-\lambda_2} \quad \text{with} \quad w_{2,ii} = 0 \quad \forall i \text{ and } w_{2,ij} = 0 \text{ if } s = t$$

A cross distance effect is not included in the present specification because this factor was found to be relatively unimportant in the SAR(1) estimation. Results of this specification[9] may be found in the last column of Table 1 and Table 2.

Apart from the travel time by auto variable which, compared to the OLS and SAR(1) estimations, is slightly more significant, the results of this formulation are not drastically different from those of the previous one. There is again a clear indication that the travel time variable should be postulated to be generic. In this specification of the error term, the destination effect is markedly smaller than the origin effect since $\hat{\lambda}_2$ is larger than $\hat{\lambda}_1$. Extending formula (13) to the SAR(2) produces predicted shares that are almost identical to the observed shares.

4 CONCLUSION

In this paper, we have extended a continuous definition of proximity used to specify the contiguity matrix of models with spatially correlated errors. This extension models unaccounted for complementarities and substitutions among origin-destination flow errors by identifying

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9 Numerous other specifications were tried but none appeared more satisfactory. Note that the estimates of ρ_1 and ρ_2 do not satisfy Minkowsky's triangle inequality condition for stability, namely that $|\rho_1| + |\rho_2| \leq 1$. The implications of this fact in spatial models seems to be even more difficult to ascertain than in time series models.

origin-based and destination-based direct and cross effects. The application to an aggregate Logit mode choice model currently in use in Winnipeg showed that significant complementarities could be identified both at the origin and the destination of work trips. Maximum likelihood estimates, obtained with normalized contiguity matrices that guarantee a stable solution, yielded results consistent with the generic interpretation of travel time. In contrast, the estimates made without taking spatial correlation of the residual errors into account, would suggest a mode-specific specification.

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APPENDIX : THE SAR(1) ESTIMATION PROCEDURE

The model that we estimate is written:

$$y = X\beta + \varepsilon$$

$$\varepsilon = \rho W(\theta)\varepsilon + u, \quad u \sim N(0,\sigma^2 I_N)$$

where the w_{ij}'s are computed according to (6) and (9). Setting $\gamma = [\rho, \theta']'$, the likelihood function is:

$$l(\beta,\sigma^2,\gamma) = (2\pi\sigma^2)^{-N/2} \mid P \mid \exp\left\{-\frac{(y - X\beta)'P'P(y - X\beta)}{2\sigma^2}\right\} \qquad (A.1)$$

where $P = I - \rho W$ and $Var(\varepsilon\varepsilon') = \sigma^2 V = \sigma^2 P^{-1} P^{-1'}$ with $V^{-1} = P'P$.

The corresponding log-likelihood function, which we maximize using the Davidon Fletcher and Powell (DFP) algorithm, is:

$$L(\beta,\sigma^2,\gamma) = -\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln(\sigma^2) + \ln \mid P \mid - \frac{(y - X\beta)'P'P(y - X\beta)}{2\sigma^2}. \qquad (A.2)$$

Applying a double concentration with respect to β and σ^2, we finally maximize:

$$L^{**}(\gamma) \propto \ln \mid P \mid - \frac{N}{2}\ln\hat{\sigma}^2, \qquad (A.3)$$

where $\hat{\sigma}^2 = (y - X\hat{\beta})'V^{-1}(y - X\hat{\beta})/N$ and $\hat{\beta} = (X'V^{-1}X)^{-1}XV^{-1}y$.

Differentiating (A.3) with respect to $\gamma_i, (i = 1,\ldots,H+1)$ leads to :

$$\frac{\partial L^{**}}{\partial \gamma_i} = \mathrm{tr}\left[P^{-1}\frac{\partial P}{\partial \gamma_i}\right] - \frac{1}{\sigma^2}(y - X\beta)'P'\frac{\partial P}{\partial \gamma_i}(y - X\beta). \qquad (A.4)$$

Formulas for $\partial P/\partial\rho$ and the $\partial P/\partial\theta_i$'s can be found in Bolduc (1985). To calculate the variance-covariance matrix of the joint vector $\hat{\eta}$ of ML estimated parameters, we are using :

$$-\left(\mathrm{E}\frac{\partial^2 L}{\partial\eta\partial\eta'}\right)^{-1}_{\eta=\hat{\eta}}.$$

The components of that matrix are:

$$\mathrm{E}\frac{\partial^2 L}{\partial\beta\partial\beta'}=-\frac{\mathrm{X}'V^{-1}\mathrm{X}}{\sigma^2},$$

$$\mathrm{E}\frac{\partial^2 L}{\partial\beta\partial\gamma_i}=\mathrm{E}\frac{\partial^2 L}{\partial\beta\partial\sigma^2}=0 \quad \forall i,$$

$$\mathrm{E}\frac{\partial^2 L}{\partial\sigma^4}=-\frac{N}{2\sigma^4},$$

$$\mathrm{E}\frac{\partial^2 L}{\partial\sigma^2\partial\gamma_i}=\frac{1}{\sigma^2}\mathrm{tr}\left[P^{-1}\frac{\partial P}{\partial\gamma_i}\right] \quad \forall i,$$

$$\mathrm{E}\frac{\partial^2 L}{\partial\gamma_i\partial\gamma_j}=-\mathrm{tr}\left[\left(\frac{\partial P}{\partial\gamma_j}P^{-1}+P^{-1}\frac{\partial P}{\partial\gamma_j}\right)\frac{\partial P}{\partial\gamma_i}P^{-1}\right] \quad \forall i,j.$$

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